

Lock-in Amplifiers for Signals Buried in Noise

By

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Phase sensitive detector followed by low pass filter is heart of lock-in amplifier. Signals 40 db below the input noise level of a microwave receiver can be recovered with lock-in technique; oscillators can be checked to high precision quickly

SIGNAL INTENSITY measurements can be made where noise would otherwise rule them out by using a lock-in amplifier. Applications to date include radio astronomy, nuclear magnetic resonance and solid state investigations.

A lock-in amplifier is essentially a narrow band detection system in which a signal is beat with a reference signal of the same frequency, giving a d-c output. The heart of the lock-in amplifier is a phase sensitive or synchronous detector, essentially a balanced mixer. The

upper side band derived from the mixer is of no interest and is stopped by a low pass filter. The lower side band (d-c) is passed by the low pass filter, the band width of which determines the band width of the amplifier. Sometimes the lock-in amplifier is operated with the reference frequency differing from the signal frequency by as little as 0.1 cps with difference frequency output recorded directly on a strip chart recorder.

While the detector elements that function as the mixer in a syn-

chronous detector are basically nonlinear, the mixer itself functions as a linear device; for purposes of transient response or noise rejection the narrow bandpass achieved in the low-pass filter is completely equivalent to a corresponding bandwidth before mixing. Thus noise rejection with this technique is excellent.

Assume a balanced mixer is followed by a filter with a pass band from 0 to 10 cps, and that the reference frequency is 1,000 cps. If the signal frequency is between

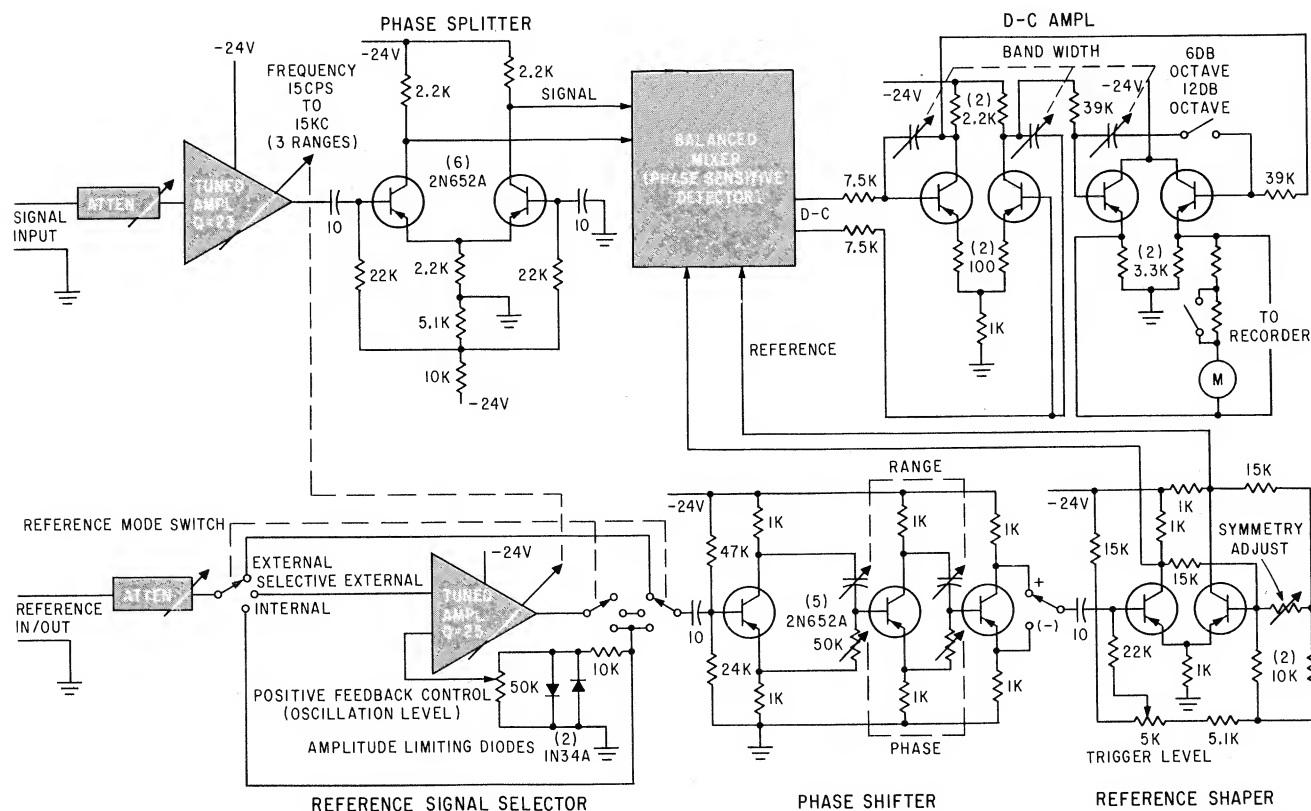


FIG. 1—Heart of lock-in amplifier is phase-sensitive detector or balanced mixer. The reference signal channel can accept an external reference or be used as a local oscillator to provide both reference and signal frequencies

990 and 1010 cps, an output signal is obtained; signals outside this band are attenuated by the low pass filter. Regardless of how the center frequency is shifted (in this example it is set at 1 Kc), the output is always the narrow band of frequencies passed by the low pass filter.

The circuit has several advantages over a conventional tuned amplifier followed by a single square law detector. First, it is easily tunable since the reference signal determines the center frequency of the pass band. Second, no matter how narrow the bandwidth of the detection system, the center of the pass band is always locked to the signal frequency if the signal is available for use as the reference. It is this characteristic that gives the lock-in amplifier its name. Since the problem of recovering a signal from noise is essentially the problem of detecting the signal with a narrow bandwidth device, the narrow band lock-in amplifier is a powerful and highly versatile tool for this purpose.

Balanced mixers are essentially nonlinear devices. Consequently, even if the reference signal $R(t)$ is sinusoidal, the function $R'(t)$ (at the same frequency but different phase) with which the signal is multiplied by the mixer in general will be nonsinusoidal. However, $R'(t)$ can be written as a Fourier series in harmonics of $R(t)$

$$R'(t) = A_0 + \sum_1^{\infty} A_n \cos n\omega t + \sum_1^{\infty} B_n \sin n\omega t \quad (1)$$

where ω is the angular frequency associated with $R(t)$. The phase of the reference signal can be chosen so that the coefficient of the $\cos\omega t$ term (A_1) in the expansion is zero, giving

$$R'(t) = A_0 + \sum_2^{\infty} A_n \cos n\omega t + \sum_1^{\infty} B_n \sin n\omega t \quad (2)$$

Assume the signal input is

$$S(t) = C \sin(\omega t + \delta) \quad (3)$$

After passing through an idealized low-pass filter with $(\sin f)/f$ frequency response and a cut off frequency much lower than ω , the output E will be

$$E_0 = \frac{1}{T} \int_{-T/2}^{T/2} R'(u) \sin(\omega u + \delta) du \quad (4)$$

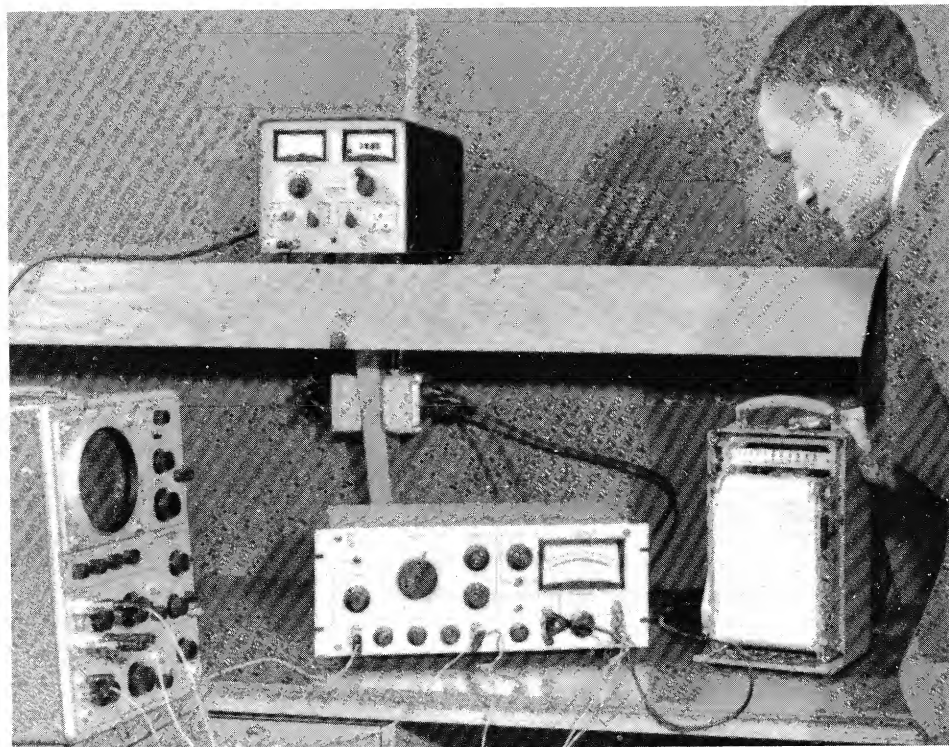
where T is the integrating time of the filter. When integrated

$$E_0 = \frac{C A_1 \cos \delta}{2} \quad (5)$$

If δ is 0 (the signal is in phase with the reference) the output is maximum and has positive polarity. If phase angle δ is 180 degrees, the output is again a maximum but of opposite polarity. If δ is 90 degrees there is no d-c output. For any phase angle the output is a function of both the amplitude and phase of the signal and the amplitude of the fundamental component of $R'(t)$. Therefore the circuit is often called a phase sensitive detector.

A phase sensitive detector is sensitive not only to a particular frequency, but also to a particular phase component of this frequency. In lock-in amplifier applications both or either property may be exploited.

The general problem is to recover a weak signal obscured by noise. Noise arises from such statistical fluctuation phenomena as Johnson



Lock-in amplifier is being used to check oscillator to high precision in short measuring interval

noise in resistors and shot noise in vacuum tubes and semiconductors; these both produce a white noise spectrum, in which the noise power per unit bandwidth is the same at all frequencies. Another source is the so-called gain modulation or flicker effect noise associated with both vacuum tube and transistor amplifying circuits. This noise frequency spectrum varies as $1/f$; a large contribution to the total noise occurs near d-c. Interference phenomena, which are not really noise but produce the same effect in obscuring the signal one desires to detect, include power line pick up and r-f interference.

With respect to white noise, little can be gained by moving the signal frequency to a different value. However, the contribution of the white noise to the output voltage of the detection system is inversely proportional to the square root of the bandwidth, and can be reduced to an arbitrary small value by reducing bandwidth. For $1/f$ noise and interference, an operating frequency different from d-c and from interfering frequencies can be selected.

Figure 1 shows a general purpose lock-in amplifier with a tuning range from 15 to 15,000 cps and a variable bandwidth down to 0.12 cps. The front panel of the instrument is shown in the photograph. The input signal, including its associated noise, is passed through a narrow-band tuned amplifier; this initial stage, though of relatively wide bandwidth, reduces

noise signals that could overdrive the phase-sensitive detector, thus allowing a larger output before non-linearity is encountered.

Furthermore, the waveform with which the signal is mixed usually contains harmonics of the reference frequency. The tuned amplifier is useful in preventing these harmonics from reaching the detector.

If the signal is only a small percent of the input to the detector, and if its peak value (signal plus noise) is within the linear range of the detector, detector output is small but can be amplified; this is accomplished by the d-c amplifier shown in Fig. 1.

In the reference channel (also Fig. 1) a variable phase shifter controls the phase between reference and signal. The phase sensitive detector used in Fig. 1 is essentially a dpdt switch requiring a square wave drive obtained by applying the reference signal to a Schmitt trigger. The waveform applied to the input of the phase shifter must be nearly sinusoidal and thus a tuned amplifier is also provided in the reference channel; The tuned amplifier can be switched out of the reference channel when a nearly sinusoidal reference signal is available. Also, the reference channel tuned amplifier can be used with a positive feedback loop as an oscillator, simultaneously driving the phase detector and providing a sinusoidal output. Thus, the phase detector can be synchronized to an external frequency or the reference frequency can be gener-

ated internally. The amplifiers are gang-tuned over the full operating range, which is 15 cps to 15 Kc in three ranges.

The lock-in amplifier has many uses. In radio astronomy it enables stellar noise signals as much as 40 db below the input noise level of the receiver to be detected and measured. It has been used to compare the frequencies of oscillators within one part in 10^6 , in only a few minutes measuring time. It is able to detect the small change in the absorbed losses of an r-f coil used in nuclear magnetic resonance experiments. It can be used as a narrow-band spectrum analyzer to detect and measure a particular Fourier component in a signal spectrum.

The weak noise signal picked up by the antenna of a radio telescope is similar to the noise generated in the radio receiver and is indistinguishable from it. Moreover, the level of the stellar noise is much lower than the effective input noise of a typical microwave receiver. A microwave radiometer, shown in Fig. 2A, designed to apply the lock-in amplifier technique to this problem was described by R. H. Dicke in 1947¹.

The input of the receiver is alternately switched between the signal from the antenna and a laboratory noise source (a warm resistor generating thermal noise). The microwave switching device is driven by a small synchronous motor and the driving frequency applied to the synchronous motor

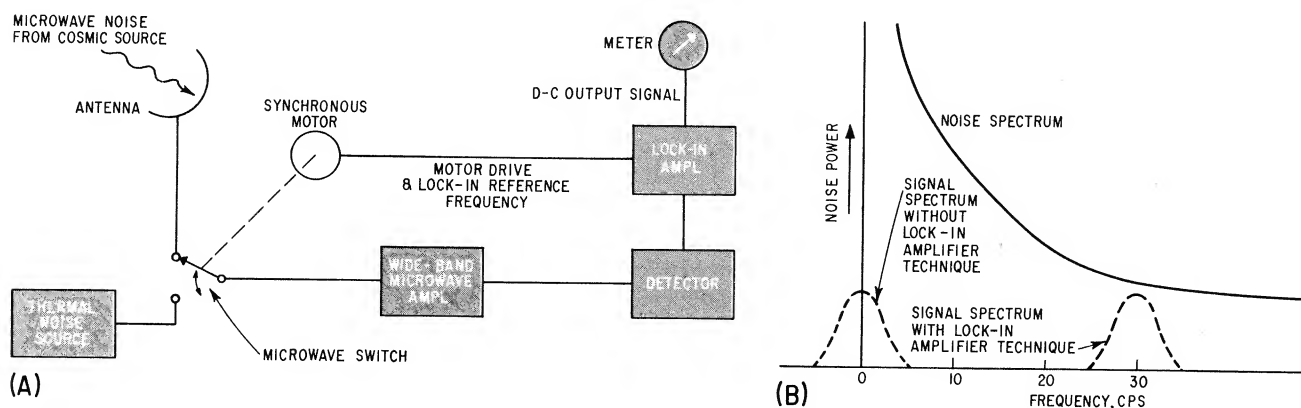


FIG. 2—Lock-in amplifier is used in radio telescope (A) to detect cosmic noise signals 40 db below the level of the wideband microwave receiver used to amplify the signals. Effect of the lock-in technique (B) is to shift the signals of interest to a less noisy part of the spectrum

is also applied to the lock-in amplifier as reference. If the level of noise from the antenna is different from the laboratory noise source, the switching produces a noise signal whose amplitude varies at the frequency used to drive the synchronous motor. This modulated noise signal is then applied to the input of the microwave receiver, which consists of a wideband amplifier followed by a detector. The output of this amplifier consists of two components: one is the modulated noise signal produced by the switch; the other, usually much larger, is the noise generated at the input of the microwave amplifier itself. This combination is applied to the detector.

Consider the output of the detector in a small interval about the frequency at which the microwave switch is driven. All the frequency components of the modulated noise signal carry sidebands corresponding to the frequency at which the switch is driven; thus all these frequency components contribute to the output of the detector. The contribution of the noise produced at the amplifier input to the output of the detector at this frequency is almost entirely due to intermodulation between the various frequency components of this noise signal. The result is, while essentially all the modulated noise signal contributes to the detector output in this frequency interval, only a small fraction of the internally generated receiver noise contributes to it. The output of the detector is then fed into the input of the lock-in amplifier.

The lock-in amplifier selects that part of the detector output lying in the frequency interval provided by the synchronous motor-driven microwave switch and converts it to a equivalent bandwidth at d-c. As has been shown, the bandwidth is determined by the low-pass filter at its output. The output of the lock-in amplifier consists of the d-c voltage due to the modulated cosmic noise signal plus fluctuations due to receiver noise. The d-c level is entirely due to, and gives a measure of, the strength of the cosmic noise source. The parameters of the system are such that

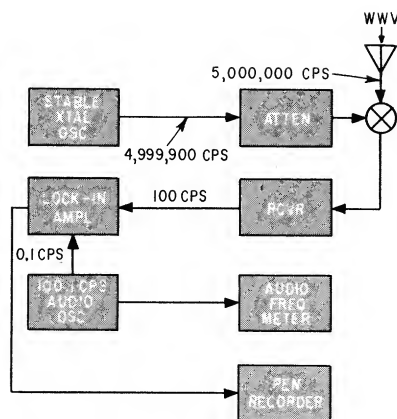


FIG. 3—Oscillators can be checked to one part in 10^{10} in five minutes using lock-in technique

cosmic noise sources that produce a level 40 db below the effective input noise level of the microwave receiver may be detected.

Another application is in the study of a particular phase component of a signal in the presence of a quadrature component. This situation occurs in making a measurement that depends on the balance of an a-c bridge. By setting the phase of the reference signal equal to the phase component of interest, the instrument will reject the quadrature component completely. In the case of balancing an a-c bridge, if one desires to deal with both the in-phase and the quadrature component, the lock-in amplifier can be used to pick up the in-phase and the quadrature components separately and allows each to be balanced separately. This usually allows the bridge to be balanced systematically and avoids the typical successive approximations approach.

An example of an application where the reference frequency is different from the signal frequency is the WWV comparison system shown in Fig. 3. The frequency of the precision crystal oscillator is offset 100 cps from WWV, to 4,999,900.0 cps if the 5-Mc carrier is used.

A small amount of the oscillator output is applied to the antenna terminals of the receiver, which is tuned to the 5-Mc signal from WWV. The result is that the 100-cps beat frequency between the oscillator and WWV appears at the audio output terminals of the

receiver; this signal is applied to the input of the lock-in amplifier. The reference frequency for the lock-in amplifier is set to 100.1 cps and the output is thus 0.1 cps, which may be recorded.

In this example the frequency of the oscillator is given by the standard WWV frequency minus the audio frequency, plus the frequency exhibited on the recorder. If a comparison is desired to one part in 10^7 (to 0.5 cps), the audio frequency need only be known to 0.5 percent and the 0.1 cps being recorded may be completely neglected provided it stays reasonably close to this value throughout the entire time of the measurement interval.

Since this technique involves measurement intervals of about twenty minutes, even inexpensive commercial audio oscillators are adequate for the check. For a high precision measurement, such as a local comparison between two stable crystal oscillators, or between a crystal oscillator and an atomic clock with an accuracy of better than one part in 10, the beat frequency should be recorded for about twenty minutes and a cycle counter and interval timer should be used to measure the time interval for about 10^5 audio frequency oscillations.

Standard commercial interval timers have a precision adequate for this application. Since the recorder allows a phase measurement at the beginning and end of the measurement interval, the accuracy of the measurement can be substantially greater than the total number of cycles in the measurement interval. If, the recorded output phase can be read to better than 0.1 radian, a comparison accurate to one part in 10^{10} can be made in an interval containing 3×10^8 cycles of crystal frequency. At one megacycle this is 300 seconds or five minutes, a short time for a measurement of such high precision.

REFERENCE

- (1) R. H. Dicke, Review of Scientific Instruments, 17, p 268, 1947.